CHALMERS UNIVERSITY OF TECHNOLOGY 2009-01-12
Microtechnology och nanoscience 14.00-18.00

Exam in FMI036, Superconductivity and low temperature physics

Teacher: Per Delsing, phone: 772 3317, 070-3088317
Allowed aids: Tefyma, Physics Handbook, Stand Math Tables and similar handbooks, calculator, and one A4 sheet with handwritten notes.
Max points: 30p
Grading: Grade 3: 5p on problem 1 and 15p in total, (Grade 4: 20p, Grade 5: 25p)
Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams.

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1. Short questions to test the understanding of concepts.
Give short descriptions or definitions (use diagrams if appropriate) of the following:

Superconductors

a) Describe the Meissner effect. How is it different in type I and type II superconductors. Draw a figure showing the magnetization.
b) What is the meaning of the coherence length in a superconductor? How can it be interpreted?
c) Sketch the BCS-density of states for excitations in an ordinary superconductor such as tin. Draw a figure and give important values.
d) Describe briefly the crystal structure of the High-Tc superconductor YBCO.

Helium

e) What is Bose-Einstein condensation and how can you make a connection to the two-fluid model?
f) Sketch the low temperature phase diagram for ^{3}He. Which are the different phases? Give the most important temperatures and pressures.
g) What is the meaning of the second sound in a superfluid? What is different from ordinary sound?

Cryogenics

h) Explain the difference between a primary thermometer and a secondary thermometer and give two examples of each type.
i) Sketch the resistivity for copper as a function of temperature, state the approximate temperature dependencies in different parts of the curve and what is the origin of the different parts.
j) How can a dilution refrigerator be used for cooling, what temperatures and cooling powers can be reached?

2. High-frequency resistance in superconductors.

a) Explain why a superconductor, which has zero resistance at dc, can have a finite resistance at high frequency? (1p)
b) Start from the two fluid model and derive an expression for the complex conductivity of a superconductor at high frequencies. The force equations for the normal fluid and the superfluid are given by:

$$m\frac{\partial\langle v_n\rangle}{\partial t} + m\frac{\langle v_n\rangle}{\tau} = -eE$$

$$m\frac{\partial v_s}{\partial t} = -eE$$

where v_n (v_s) is the velocity of the normal- (super-) fluid. (3p)

3. BCS theory

The BCS ground state is given by $|BCS\rangle = \prod_k \left(u_k + e^{i\theta} v_k c^\dagger_{k+s,\uparrow} c^\dagger_{-k+s,\downarrow}\right)|0\rangle$

a) Explain in simple terms how the electron phonon interaction can give rise to an attractive interaction between electrons that leads to (Cooper-)pairing. (1p)

b) Explain the different parts of the ground state. What is |0> and θ. What does the operators $c^\dagger$ and c do? What does u_k and v_k describe, explain with a figure how they vary with energy? (3p)

4. Tunneling in a Superconductor-Insulator-Normal metal (SIN) tunnel junction.

a) Set up the integral expression for the current in a SIN junction at finite temperature, include both the left and the right going currents and simplify the expression. Draw an energy diagram for the tunnel junction and explain the used variables. (2p)

b) Calculate the current-voltage characteristics at T=0 for both positive and negative voltages and make a drawing of the current-voltage characteristics indicating voltage and current scales. (2p)

5. Superfluid Helium

a) What is the meaning of the critical velocity in a superfluid? Draw the dispersion relation for excitations in ^{4}He and explain what different types of excitations you can get. (2p)

b) Derive an expression for the critical velocity to create excitations in the superfluid. Calculate the critical velocity of superfluid flow in liquid ^{4}He, assuming that the only excitations from the condensate has a dispersion law $E = \Delta + (p-p_0)^2/(2m^*)$, where $\Delta/k_B \approx 8.65$ K, $p_0/h \approx 1.92$Å^{-1} and the effective mass $m^* \approx 0.16 \cdot m_4 \approx 0.16 \cdot 6.69 \cdot 10^{-27}$ kg? (2p)

6. Low temperature techniques.

The cooling power of a dilution refrigerator is given by $P = a\cdot(T^2 - T^2_{\min})$, with $a = 12$ [mW/K^2], and $T_{\min}$=10 mK is the temperature of the mixing chamber without any heat load. A mechanical manipulator made of stainless steel in the shape of a rod and with the dimensions L=0.50 m and diameter 1.0 mm, is mounted between the IVC-flange (T=4.2K) and the mixing chamber. The heat conductivity of stainless steel is $\kappa = 0.15 \cdot T$ W/(m·K^2).

a) Calculate the power with which the manipulator heats the mixing chamber. (1.5p)

b) What is the new minimum temperature with the manipulator installed. (1p)

c) What minimum temperature do you get if you can heat sink the rod at 1.6K with an effective length of 0.30 m instead. (1.5p)

Good luck !