CHALMERS UNIVERSITY OF TECHNOLOGY 2007-03-15
Mikrotechnology och nanoscience Kl: 08.30-12.30
Building V

Exam in FMI036, Superconductivity and low temperature physics

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Allowed aids: Tefyma, Physics Handbook, Stand Math Tables and similar handbooks, calculator, and one A4 sheet with handwritten notes.
Max points: 30p
Grading: Grade 3: 5p on problem 1 and 15p in total, (Grade 4: 20p, Grade 5: 25p)
Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

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1. Short questions to test the understanding of concepts.
Give short answers, descriptions, or definitions (use diagrams if appropriate) of the following:

a) How does the thermodynamical critical field H_C relate to the energy gap Δ of a superconductor.
b) What is the isotope effect for superconductors, how can it be explained?
c) What is the meaning of the parameters α and β in the Ginzburg Landau theory.
d) Mention two important applications of superconductivity, shortly describe the advantages over competing technologies.
e) Sketch the doping phase diagram for YBCO (T vs. hole doping). Describe the different phases.

f) Sketch the low temperature phase diagram (P vs T) for ^{3}He. Which are the different phases ? Give the most important temperatures and pressures.
g) What is Bose Einstein condensation and how can you make a connection to the two fluid model ?

h) Sketch the resistivity for copper as a function of temperature, state the approximate temperature dependencies in different part of the curve and what is the origin of the different parts.
i) Describe briefly how a Joule-Thompson cooler works. What is the meaning of the inversion temperature ?
j) How can a superconductor be used as a heat switch (for example in an adiabatic demagnetization refrigerator) at low temperature ?

(1p per question)

2. The London equations.

a) Derive the expression $\Lambda \cdot \bar{J}_S = -\left(\frac{\hbar}{2e}\bar{\nabla}\theta + \bar{A}\right)$. Start from the second Ginzburg-Landau equation, $\bar{J}_S = \frac{ie\hbar}{2m}\left(\Psi^*\bar{\nabla}\Psi - \Psi\bar{\nabla}\Psi^*\right) - \frac{2e^2}{m}\bar{A}\Psi^*\Psi$, assuming that the order parameter is $\Psi = \sqrt{n_p} \cdot e^{i\theta}$. Give an expression for the London parameter Λ. (2p)
b) Derive the first London equation from the definition of the supercurrent density $J_S = -2en_p v_S$ (1p)
c) Derive the second London equation from the expression derived in a) (1p)

3. Tunneling.

a) Derive the expression for the Josephson inductance from the two Josephson relations. (1p)
c) Derive the current-voltage characteristics of an SIN tunnel junction at zero temperature. Explain what happens at elevated temperature. (3p)

4. Flux quantization

a) Show mathematically that the magnetic flux is quantized in a thick superconducting ring. How big is the quantized flux ? (1p)
b) What is an Abrikosov vortex, describe how the magnetic flux density B and the magnitude of the order parameter $|\Psi|$ varies with the distance from the vortex center. (1p)

5. Cryogenics

A resistive thermometer is attached to the mixing chamber of a dilution refrigerator. To insulate the thermometer electrically, it is attached to the mixing chamber via a copper plate which in turn is placed on sapphire sheet. The thermal conductivity of sapphire is given by $\kappa(T) = 3.2 \cdot T^3$ (W/mK^4), the sheet is 1cm^2 in area and 1mm thick.

The temperature measurement will inevitable heat up the thermometer. How much power can you use to do the measurement if you want to keep the temperature difference between the thermometer and the mixing chamber smaller than 1mK. Assume that the mixing chamber is at 20 mK (independent of the power) and that the thermal conductance is dominated by that of the saphire sheet. (2p)

6. Superfluid Helium

a) Describe what is meant by "critical velocity" in superfluid helium. (1p)

b) Derive an expression for how to find the critical velocity if you know the dispersion relation E(p) for excitations in super fluid helium. (1.5p)

c) Describe the different symmetry breaking mechanisms that can be found in superfluid ^{3}He and explain how the superfluid A and the B phases relate to these mechanisms. (1.5p)

7. BCS theory

a) What is the meaning of u_k and v_k in the BCS theory and how are they related to each other. Sketch the dependence of v_k^2 as a function of k-value for the BCS ground state. How does this ground state differ from the Fermi ground state of a normal metal at zero temperature. (1p)

b) For a weakly coupled superconductor we can assume that the energy gap Δ is independent of k and that it can be written as $\Delta = \sum_k V_k u_k v_k$. u_k is given by $u_k^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + \Delta^2}}\right)$ where ε_k is the energy of an electron with the momentum $\hbar k$ (measured relative to the Fermi energy). V_k is the interaction energy between the two electrons of a Cooper-pair, and is due to electron phonon interaction. It is considered to be constant, V, for energies smaller than the Debey energy $|\varepsilon_k| < \hbar\omega_D$, and zero for larger energies. Convert the sum to an integral and derive an expression for the energy gap Δ as a function of V. (3p)

GOOD LUCK !