Elektriska Nät och System

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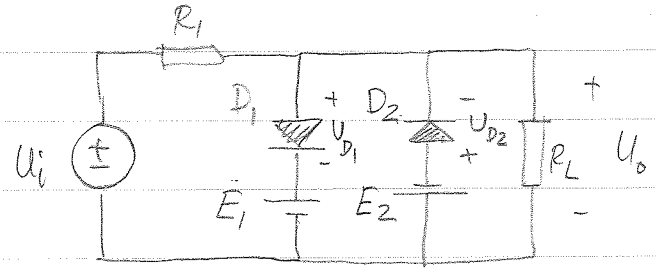
1/



KVL: $U_{D1} + E_1 - U_o = 0$

$U_{D1} = U_o - E_1$

D_1 leder ström om $U_o > E_1$

KVL $U_{D2} + U_o + E_2 = 0$

$U_{D2} = -(U_o + E_2)$

D_2 leder ström om $U_o + E_2 < 0 \Rightarrow U_o < -E_2 = -4{,}0\ V$

Fall 1 D_1, D_2 spärrar $-E_2 < U_o < E_1$, $-4{,}0 < U_o < 6{,}0\ V$

$U_o = U_i \dfrac{R_L}{R_1 + R_L} = U_i \cdot \dfrac{1}{2}$

Fall 2 D_1 leder, D_2 spärrar: , $U_o = E_1 = 6{,}0 V$, inträffar då $U_i > 12 V$

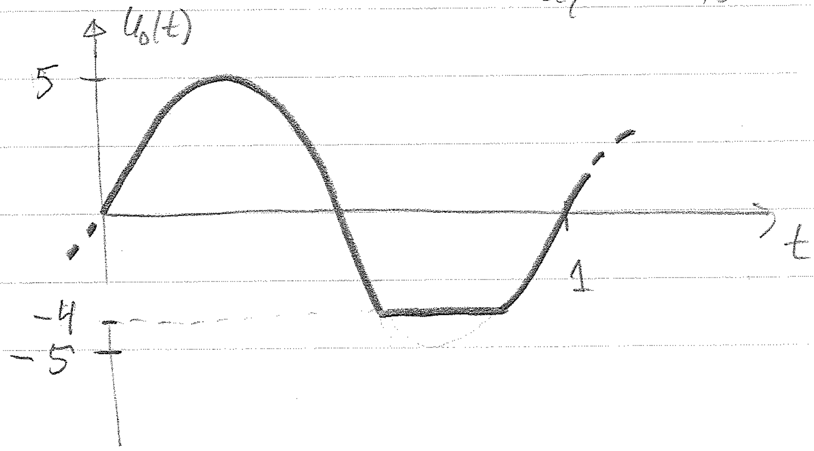
Fall 3 D_1 spärrar, D_2 leder: $U_o = -E_2 = -4{,}0 V$, inträffar då $U_i < -8{,}0 V$

$u_i(t) = 10 \sin(2\pi t)\ V$

$\omega = 2\pi\ r/s$

$f = 1\ Hz$

$T = 1/f = 1\ s$



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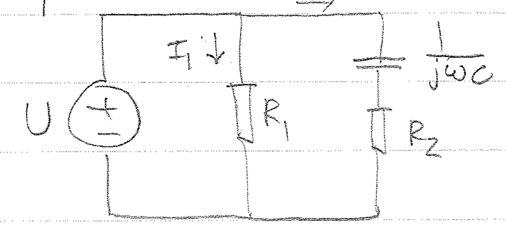
2./

jω-transf.



$u(t) = 15\sqrt{2}\,\sin(\omega t)$

$\omega = 5000$ r/s

$$\frac{1}{\omega C} = \frac{1}{5000 \cdot 10^{-6}} = 200$$

$R_1 = 100\ \Omega$, $R_2 = 60\ \Omega$, $C = 1\,\mu F$

$$I = \frac{U}{R_2 + \frac{1}{j\omega C}} = \frac{15\sqrt{2}}{60 - j200}$$

$$I_1 = \frac{U}{R_1} = \frac{15\sqrt{2}}{100}$$

$$I = \frac{15\sqrt{2}\,(60 + j200)}{60^2 + 200^2} = \frac{15\sqrt{2} \cdot 10}{43600}(6 + j20) = \frac{15\sqrt{2}}{4360}\sqrt{436}\,\angle 73{,}3^\circ =$$

$$= 0{,}10\,\angle 73{,}3^\circ \Rightarrow i(t) = 0{,}10\,\sin(\omega t + 73{,}3^\circ)\ \text{A}$$

Komplex effekt som källan avger ("ström ut från plus")

$$S = \frac{1}{2} U (I_1 + I)^* = \frac{1}{2} 15\sqrt{2} \left(\frac{15\sqrt{2}}{100} + \frac{15\sqrt{2} \cdot 10}{43600}(6 - j20) \right) =$$

$$= \frac{1}{2} \frac{(15\sqrt{2})^2}{100} \left[1 + \frac{10}{436}(6 - j20) \right] = \frac{225}{100} \left(1 + \frac{60}{436} - j\frac{200}{436} \right) =$$

$$= 2{,}25(1{,}14 - j0{,}46) = 2{,}56 - j1{,}03 = 2{,}76\,\angle -22{,}0\ \text{VA}$$

$S = P + jQ$

Kommentar: u(t) en sinus signal – jω metoden "utgår" ifrån cosinus. Svaret blir samma om u(t) skrivs om med cosinus

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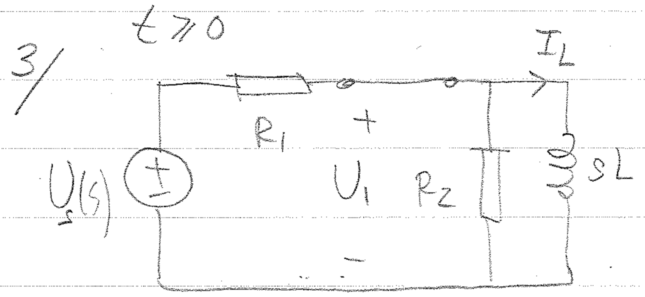
3/ $t \geq 0$



Laplace transformera!
Ingen beg. energi

$$u_s(t) = 30\cos 5t \overset{\mathcal{L}}{\longleftrightarrow} 30\frac{s}{s^2+25}$$

$$U_1 = U_s \frac{R_2 // sL}{R_1 + R_2 // sL} = U_s \frac{\frac{sR_2L}{R_2+sL}}{R_1 + \frac{sR_2L}{R_2+sL}} = U_s \frac{sR_2L}{R_1R_2 + sR_1L + sR_2L}$$

$$I_L = \frac{U_1}{sL} = U_s \frac{R_2}{R_1R_2 + sL(R_1+R_2)} = U_s \frac{\frac{R_2}{L(R_1+R_2)}}{s + \frac{R_1R_2}{L(R_1+R_2)}} =$$

$$= U_s \cdot \frac{20/(60+20)}{s + \frac{60 \cdot 20}{1\cdot(60+20)}} = U_s \cdot \frac{0{,}25}{s+15} = 30 \cdot \frac{s \cdot 0{,}25}{(s^2+25)(s+15)}$$

$$= \frac{7{,}5\,s}{(s^2+25)(s+15)} = \left\{ \begin{array}{l} \text{Partialbr.} \\ \text{uppdela} \end{array} \right\} = \frac{As+B}{s^2+25} + \frac{C}{s+15}$$

$$7{,}5\,s = (As+B)(s+15) + C(s^2+25)$$

$$\left.\begin{array}{ll} s^2: & 0 = A + C \\ s^1: & 7{,}5 = 15A + B \\ s^0: & 0 = 15B + 25C \end{array}\right\} \text{Lös ekv. systemet} \Rightarrow \begin{array}{l} A = 0{,}45 \\ B = 0{,}75 \\ C = -0{,}45 \end{array}$$

$$I_L = \frac{0{,}45\,s}{s^2+25} + 0{,}75 \cdot \frac{1}{5} \cdot \frac{5}{s^2+25} - \frac{0{,}45}{s+15}$$

Inv. Laplacetransf.

$$i_L(t) = \left(0{,}45\cos 5t + 0{,}15\sin 5t - 0{,}45e^{-15t}\right) \cdot \theta(t) \quad \text{A}$$

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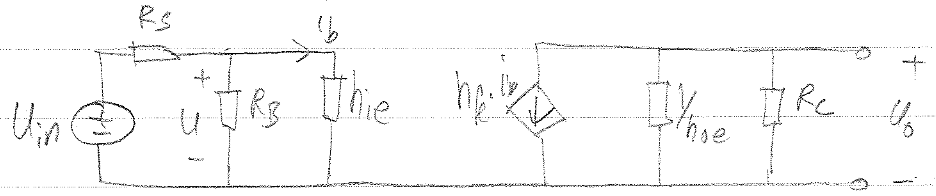
4. Småsignalschema



$$\begin{cases} u = u_{in} \dfrac{R_B /\!/ h_{ie}}{R_S + R_B /\!/ h_{ie}} \Rightarrow u_{in} = u \cdot \dfrac{R_S + R_B /\!/ h_{ie}}{R_B /\!/ h_{ie}} \\ u = i_b \cdot h_{ie} \\ u_o = -h_{fe} \cdot i_b \cdot \left(\dfrac{1}{h_{oe}}\right) /\!/ R_C = -h_{fe} \cdot \dfrac{u}{h_{ie}} \cdot \dfrac{\frac{1}{h_{oe}} \cdot R_C}{\frac{1}{h_{oe}} + R_C} = -u \dfrac{h_{fe}}{h_{ie}} \cdot \dfrac{R_C}{1 + R_C \cdot h_{oe}} \end{cases}$$

$$\frac{u_o}{u_{in}} = -\frac{u \cdot \frac{h_{fe}}{h_{ie}} \cdot \frac{R_C}{1 + R_C \cdot h_{oe}}}{u \cdot \dfrac{R_S + \frac{R_B h_{ie}}{R_B + h_{ie}}}{\frac{R_B h_{ie}}{R_B + h_{ie}}}} = -\frac{h_{fe}}{\cancel{h_{ie}}} \cdot \frac{R_C}{(1 + R_C h_{oe})} \cdot \frac{R_B \cdot \cancel{h_{ie}}}{[R_S(R_B + h_{ie}) + R_B h_{ie}]}$$

$$\frac{u_o}{u_{in}} = -\frac{150 \cdot 500 \cdot 27 \cdot 10^3}{\underbrace{(1 + 500 \cdot 48 \cdot 10^{-6})}_{1{,}024} \underbrace{[(27 + 0{,}52) \cdot 10^3 + 27 \cdot 520] \cdot 10^3}_{41560}} = -47{,}6 \text{ ggr}$$

Svar: $\dfrac{u_o}{u_{in}} = -47{,}6$ ggr

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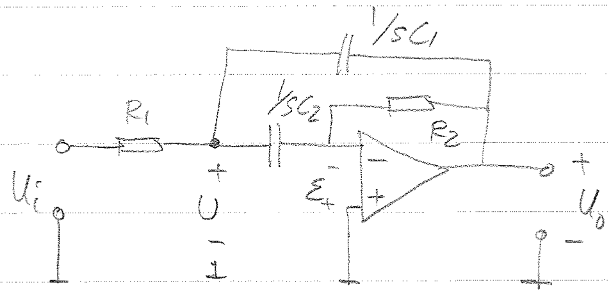
5.



$$\left.\begin{array}{l}\text{Ideal Op-först.}\\ \text{Neg. återk.}\end{array}\right\} \Rightarrow \varepsilon = 0$$

$i_{op} = 0$

$$\text{KCL:}\quad \begin{cases} \dfrac{U_i - U}{R_1} + (U_0 - U)sC_1 + \dfrac{U_0}{R_2} = 0 \\ UsC_2 + \dfrac{U_0}{R_2} = 0 \end{cases} \Rightarrow \quad \begin{array}{l} \dfrac{U_i}{R_1} = -U_0\left(sC_1 + \dfrac{1}{R_2}\right) + U\left(\dfrac{1}{R_1} + sC_1\right) \\ U = -\dfrac{U_0}{sR_2C_2} \end{array}$$

$$\frac{U_i}{R_1} = -U_0\left[\frac{1+sR_2C_1}{R_2} + \frac{1+sR_1C_1}{R_1}\cdot\frac{1}{sR_2C_2}\right]$$

$$\frac{U_i}{U_0} = -\left(\frac{R_1}{R_2}(1+sR_2C_1) + \frac{1+sR_1C_1}{sR_2C_2}\right) = -\frac{1}{R_2}\left(R_1 + sR_1R_2C_1 + \frac{1+sR_1C_1}{sC_2}\right)$$

$$= -\frac{sR_1C_2 + s^2R_1R_2C_1C_2 + 1 + sR_1C_1}{sR_2C_2} =$$

$$= -\frac{s^2R_1R_2C_1C_2 + sR_1(C_1+C_2) + 1}{sR_2C_2}$$

$$\frac{U_0}{U_i} = -\frac{s/R_1C_1}{s^2 + s\dfrac{1}{R_2}\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}\right) + \dfrac{1}{R_1R_2C_1C_2}} = -\frac{sA}{s^2 + sB + \omega_0^2}$$

Svar: Bandpassfilter

$$\text{Max först} = \left|\frac{A}{B}\right| = \frac{1}{R_1C_1}\cdot\frac{R_2C_1C_2}{(C_1+C_2)} = \frac{R_2}{R_1}\cdot\frac{C_2}{(C_1+C_2)}$$

6,

$$F(s) = \left(\frac{10}{1 + \frac{s}{\omega_1}} \right)^3 \qquad \omega_1 = 10^4 \ r/s$$

Studera $\beta F(j\omega)$ $\qquad \beta \in \mathbb{R}$

Gränsfall för instabilitet, $\beta F(j\omega) = -1$

$$\beta F(j\omega) = \frac{\beta 10^3}{\left(1 + j\frac{\omega}{\omega_1}\right)^3} = -1$$

$$-\beta 10^3 = \left(1 + j\frac{\omega}{\omega_1}\right)^3 = \left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + 2j\frac{\omega}{\omega_1} - \frac{\omega^2}{\omega_1^2}\right) =$$

$$= 1 + j\frac{2\omega}{\omega_1} - \frac{\omega^2}{\omega_1^2} + j\frac{\omega}{\omega_1} - 2\frac{\omega^2}{\omega_1^2} - j\frac{\omega^3}{\omega_1^3}$$

$$\{\mathrm{Im}\}: \quad \frac{2\omega}{\omega_1} + \frac{\omega}{\omega_1} - \frac{\omega^3}{\omega_1^3} = 0 \Rightarrow 3 = \frac{\omega^2}{\omega_1^2} \qquad \omega = \sqrt{3}\,\omega_1$$

$$\{\mathrm{Re}\}: \quad -\beta 10^3 = 1 - \frac{3\omega^2}{\omega_1} = \{\omega = \sqrt{3}\,\omega_1\} = 1 - 9 = -8 \Rightarrow \beta = 8 \cdot 10^{-3}$$

Amplitudmarginal 6dB:

$$|\beta_1 F(j\omega)| = X \text{ då } \angle \beta_1 F(j\omega) = -180^\circ \qquad \omega = \sqrt{3}\,\omega_1$$

$$20 \cdot {}^{10}\log X = -6 \Rightarrow X = 10^{-\frac{6}{20}} = 0{,}50$$

$$|\beta_1 F(j\omega)|_{\omega = \sqrt{3}\omega_1} = 0{,}5 \Rightarrow \frac{\beta_1 10^3}{\left(\sqrt{1 + (\sqrt{3})^2}\right)^3} = 0{,}5$$

$$\beta_1 = \frac{0{,}5 \cdot 10^3}{\left(\sqrt{1+3}\right)^3} = \frac{0{,}5 \cdot 10^3}{8} = \frac{\beta}{2} = 4 \cdot 10^{-3}$$

Svar:

a) $\beta < 8 \cdot 10^{-3}$

b) $\beta_1 = 4 \cdot 10^{-3}$

Elektriska Nät och System, F2

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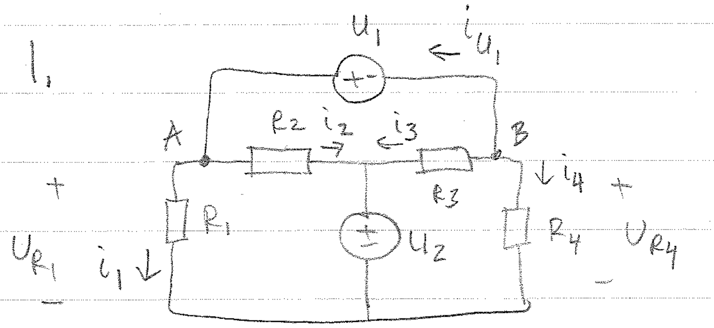
1.



$u_1 = 6{,}0\,V$

$u_2 = 4{,}0\,V$

$R_1 = R_2 = 2{,}0\,k\Omega$

$R_3 = R_4 = 4{,}0\,k\Omega$

$$\left.\begin{aligned} KCL_A &: \quad i_1 + i_2 - i_{u_1} = 0 \\ KCL_B &: \quad i_3 + i_4 + i_{u_1} = 0 \end{aligned}\right\} i_1 + i_2 + i_3 + i_4 = 0$$

$$i_1 = \frac{U_{R_1}}{R_1} \; ; \; i_4 = \frac{U_{R_4}}{R_4} \; ; \; i_2 = \frac{U_{R_1} - U_2}{R_2} \; ; \; i_3 = \frac{U_{R_4} - U_2}{R_3}$$

$$\begin{cases} \dfrac{U_{R_1}}{R_1} + \dfrac{U_{R_1} - U_2}{R_2} + \dfrac{U_{R_4} - U_2}{R_3} + \dfrac{U_{R_4}}{R_4} = 0 \\ U_{R_1} = U_1 + U_{R_4} \end{cases}$$

$$(U_1 + U_{R_4})\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + U_{R_4}\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = U_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$U_{R_4}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = U_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - U_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$U_{R_4} = \frac{U_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - U_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{4\left(\frac{1}{2} + \frac{1}{4}\right) - 6\left(\frac{1}{2} + \frac{1}{2}\right)}{\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4}} =$$

$$= \frac{4 \cdot \frac{3}{4} - \frac{24}{4}}{\frac{6}{4}} = -\frac{12}{6} = -2\,V$$

$$i_4 = \frac{U_{R_4}}{R_4} = \frac{-2}{4 \cdot 10^3} = -\frac{1}{2}\,mA$$

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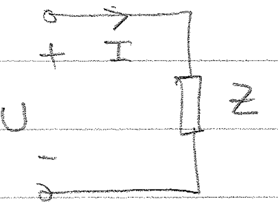
$j\omega$-transformera

2.



$|U| = 880\sqrt{2}$ V

$P = 3{,}0$ kW

$Q = 4{,}0$ kVAr

$$\begin{cases} S = P + jQ = \frac{1}{2} U I^* \\ U = Z \cdot I \end{cases} \Rightarrow \quad I = \frac{U}{Z}, \quad I^* = \frac{U^*}{Z^*}$$

$$P + jQ = \frac{1}{2}\frac{UU^*}{Z^*} = \frac{1}{2}\frac{|U|^2}{Z^*}$$

$$Z^* = \frac{1}{2}\frac{|U|^2}{P+jQ} = \frac{1}{2}\frac{|U|^2(P-jQ)}{P^2+Q^2} \quad \text{och}$$

$$Z = \frac{|U|^2}{2(P^2+Q^2)}(P+jQ) = \frac{880^2 \cdot 2}{2(3000^2+4000^2)}(3000+j4000) =$$

$$= 92{,}9 + j124 =$$

$$= 154.9 \angle 53{,}1^\circ \quad \Omega$$

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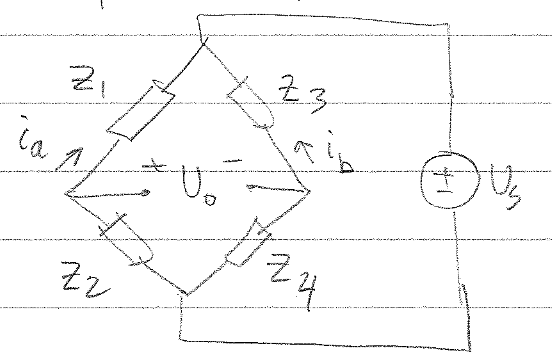
3. $j\omega$ - transformera



$$Z_1 = R_1 \| C_1 = \frac{R_1 \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

$$Z_2 = R_2 \quad ; \quad Z_3 = R_3$$

$$Z_x = R_x + j\omega L_x = Z_4$$

$U_0 = 0$

$$\text{KVL: } \left.\begin{aligned} i_a Z_1 - i_b Z_3 &= 0 \\ -i_a Z_2 + i_b Z_4 &= 0 \end{aligned}\right\} \qquad \left.\begin{aligned} i_a Z_1 &= i_b Z_3 \\ i_a Z_2 &= i_b Z_4 \end{aligned}\right\} \Rightarrow \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

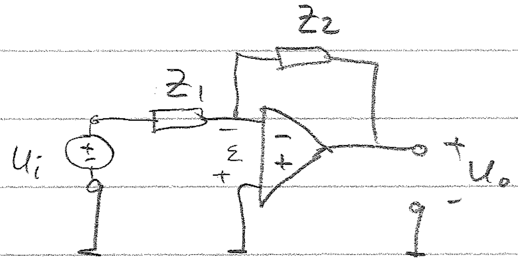
$$Z_x = Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{R_2 R_3}{R_1}(1 + j\omega R_1 C_1) =$$

$$= \frac{R_2 R_3}{R_1} + j\omega R_2 R_3 C_1 = R_x + j\omega L_x$$

Svar: $R_x = \dfrac{R_2 R_3}{R_1}$

$L_x = R_2 R_3 C_1$

4,



$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

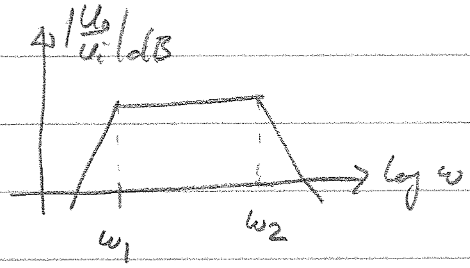
$$Z_2 = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

Ideal OP-först. + Neg. återkoppling $\Rightarrow \varepsilon = 0$

KCL $\quad \frac{u_i}{Z_1} + \frac{u_0}{Z_2} = 0 \qquad \frac{u_0}{u_i} = -\frac{Z_2}{Z_1}$

$$\frac{u_0}{u_i} = -\frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)} = -\frac{j\omega R_2 C_1}{\left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + j\frac{\omega}{\omega_2}\right)}$$

Bode diagram (antag $\omega_1 \ll \omega_2$)



- Övre gränsvinkelfrekvens

$$\omega_2 = \frac{1}{R_2 C_2} = 10^4 \text{ r/s}$$

$$C_2 = \frac{1}{R_2 \cdot 10^4} = \frac{1}{20 \cdot 10^3 \cdot 10^4} = 5{,}0 \cdot 10^{-9} = 5{,}0 \text{ nF}$$

- Pulsfall $\quad P_{rel} = \Delta t \cdot \omega_1 \cdot 100 = 2 \; [\%] \qquad \Delta t = 0{,}20 \text{ ms}$

$$\Delta t \cdot \frac{1}{R_1 C_1} = 0{,}02 \; ; \; C_1 = \frac{\Delta t}{R_1 \cdot 0{,}02} = \frac{0{,}20 \cdot 10^{-3}}{20 \cdot 10^3 \cdot 0{,}02} = 5 \cdot 10^{-7} =$$

$$= 0{,}50 \; \mu F$$

$\omega_1 = \frac{1}{R_1 C_1} = 100$ r/s $\qquad$ ∴ $\omega_1 \ll \omega_2$ Lågfrekvens och högfrekv. egenskaper påverkar ej varandra!

Svar: $C_1 = 0{,}50 \; \mu F$
$C_2 = 5{,}0 \text{ nF}$

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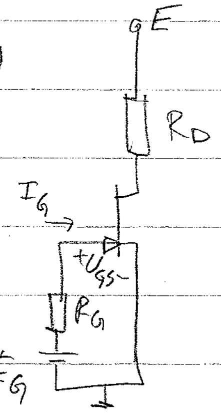
5.

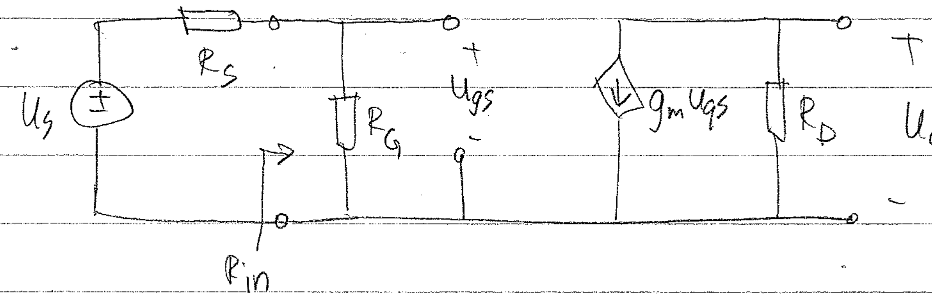
Storsignal



$I_G = 0$

$U_{GS} = E_G = -1\ \text{V}$

Småsignal



$$\begin{cases} U_{gs} = U_S \dfrac{R_G}{R_S + R_G} \Rightarrow U_S = U_{gs} \dfrac{R_S + R_G}{R_G} \\ U_o = -g_m U_{gs} R_D \end{cases}$$

$$\frac{U_o}{U_S} = -\frac{R_G}{R_S + R_G} \cdot g_m R_D$$

$$g_m = \frac{\partial i_D}{\partial u_{GS}} = \frac{\partial}{\partial u_{GS}} \left(I_{DSS} \left(1 - \frac{u_{GS}}{U_P} \right)^2 \right) = -\frac{2 I_{DSS}}{U_P} \left(1 - \frac{u_{GS}}{U_P} \right) =$$

$$= -\frac{2 \cdot 5 \cdot 10^{-3}}{-3} \left(1 - \frac{1}{3} \right) = \frac{10}{3} \cdot \frac{2}{3} \cdot 10^{-3} = \frac{20}{9} \cdot 10^{-3}$$

$$\frac{U_o}{U_S} = -\frac{100}{10 + 100} \cdot \frac{20}{9} \cdot 2 = -4{,}0$$

$R_{in} = R_G$

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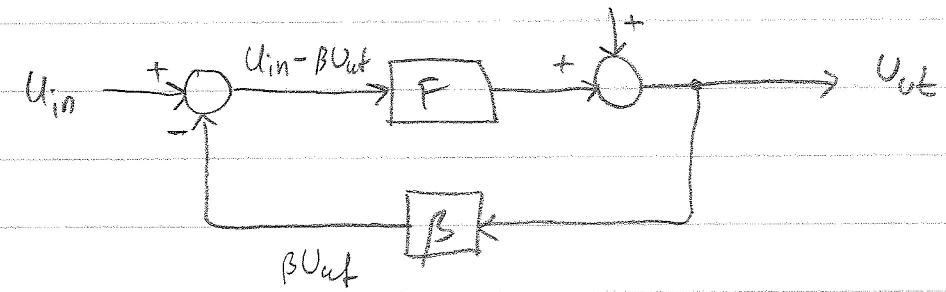
6,



$$U_{ut} = U_s + F(U_{in} - \beta U_{ut})$$

$$U_{ut}(1+\beta F) = F U_{in} + U_s$$

$$U_{ut} = U_{in} \frac{F}{1+\beta F} + \frac{U_s}{1+\beta F}$$

Vanligen är $|1+\beta F| >> 1$

Störsignal U_s dämpas med faktorn $1+\beta F$

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1/ $P_A = -500 \text{ W}$ $Q_A = 866 \text{ VAr}$

$P_B = -P_A$ $Q_B = -Q_A$

Krets A avger aktiv effekt, Krets B upptar aktiv effekt

2/ $C_1 = 160 \text{ nF}$, $C_2 = 40 \text{ nF}$

3/ a/ $i_L = 2{,}0 \text{ mA}$ $u_o = 5 \text{ V}$ OK

b/ $i_L = 2{,}0 \text{ mA}$ $u_o = 8.0 \text{ V}$ OK

c/ u_o "bottnar" $u_o = 9{,}0 \text{ V}$ $\varepsilon \neq 0$ $i_L = 1{,}125 \text{ mA}$

4/ $$u_o(t) = \frac{10}{9}\left(e^{-1000t} - e^{-100t}\right) \cdot \theta(t)$$

5/ $$\frac{v_o}{v_i} = \frac{g_m \cdot R_L /\!/ R_S}{1 + g_m R_L /\!/ R_S} = \ldots = 0{,}91$$

$(R_S = 9{,}53 \text{ k}\Omega \quad , \quad g_m = 2 \cdot 10^{-3} \text{ A/V})$

6. $$\angle \beta F = -\left(\arctan\frac{\omega}{\omega_1} + \arctan\frac{\omega}{\omega_2} + \arctan\frac{\omega}{\omega_3}\right) = -150^\circ$$

Passningsräkning ger $\omega = 2.2\,\omega_1$

$$|\beta F|_{\omega = 2{,}2\omega_1} = 1 \Rightarrow \beta = \beta_0 = 0{,}046$$

$$F_{tot} = \left.\frac{F}{1+\beta F}\right|_{\omega \to 0} = -18.3$$