Chalmers Tekniska Högskola
Institutionen för signaler och system
Avdelningen för reglerteknik

Tentamen i Reglerteknik SSY310/ERE091

October 9 th, 2020 / English version

1. Time: 4 hours. You need to be remotely supervised and ID-checked via Zoom. You need to have a computer/device with camera and microphone.
2. Teacher: (ZOOM, Skype: kulcsar@chalmers.se), Examiner: Balazs Kulcsar
3. 20 points can be reached in total (resolution 0.5 points). Table 1 shows the limits for each grade.

Tabell 1: Grades

Points	Grade
≤ 9.5	Failed
$10 \ldots 12.5$	3
$13 \ldots 15.5$	4
$16 \ldots 20$	5

4. Random sets of exam questions may randomly be assigned to students.
5. Handwritten solutions are requested (name and cid on each pages). When done, scan/photograph your solutions and compile it into one pdf document. Upload your file to Canvas (submission site closes 30 min after the examination). If the electronic version of the solution is not readable from the file, it will not be assessed (with 0 point).
6. Cooperation with or external help from other person is prohibited during examination! If signs of cooperations/external help are being discovered, *all exams involved will automatically be disqualified* and we will *automatically* report the *case to Chalmers with a request for suspension at Swedish Higher Education Authority.*
7. We may call students for oral post-check of solutions in the exam period (within a few days after the exam). Then, students will be asked (with short notice) to explain their solutions or answers.
8. All other aids can be used (books, notes, Matlab, etc.).
9. Teacher(s) will online be available for questions twice, approximately 1 hour after the start and 1 hour before the end of the exam. Examination results will be advertised approx 10 days after the exam. Inspection of results via Zoom.

Good luck!

Questions

1. Briefly answer the questions below.

   a) An LTI system has SISO and BIBO property at the same time. What can you conclude on that system? (briefly motivate your answer)? (**1 point**)

   This is input-output stable single input single output system.

   b) Compare input-output and state space modeling concepts for LTI systems. (**1 point**)

   With $n > m$ IO model defines an nth order ODE whilst the state space model uses a vector differential equation with a single order. IO vs internal aspects.

   c) Define the class of non-minimum phase systems, and give an example. (**1 point**)

   Phase changes more than the defined minimum amount $\pi(n-m)$. A pure time delay, zeros at RHP.

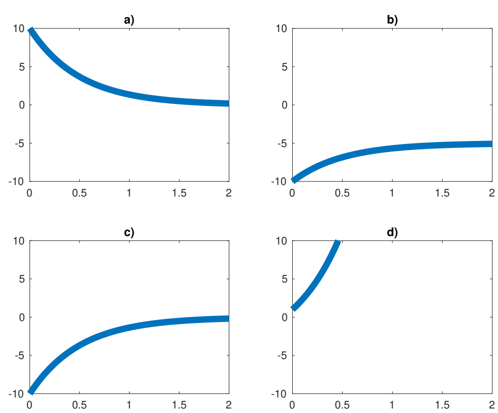
   d) What is a LAG compensator? Why to use it? (**1 point**)

   Realistic PI controller to compensate the weakness of the ideal PI term, having an unstable controller at steady state

   e) Define and interpret the terms in the cost function behind Linear Quadratic Regulator (**1 point**)

   Generalized internal (state quadratic term) and inlet (weighted input) energy minimization problem

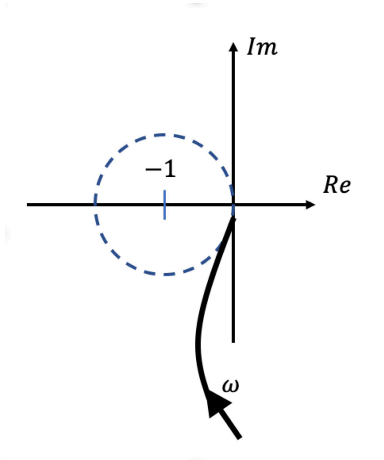
2. In Figure 1 depicts 4 step responses. Given $G(s) = \frac{\beta s}{s+\alpha}$ with finite valued $\alpha, \beta \in \mathbb{R}$. We know that the system model is asymptotically stable. Which of the plots in Fig 1 can belong to the transfer function model? (**1.5 point**)



Figur 1: Step response plots for $G(s)$

There are 2 solutions, $a)$ and $c)$ (stable, and differentiator).

3. Given a loop frequency function $L(i\omega)$. It has a Nyquist plot that is always outside of the unity circle around the real -1 point, see Figure 2. Is it describing a stable closed-loop behaviour (no poles of the open-loop model at RHP)? What is the smallest possible value for the phase margin?(**1.5 point**)



Figur 2: Nyquist plot of $L(i\omega)$

Stable by means of Nyquist. The smallest guaranteed $\phi_m = 60$ degrees (LQR has this property!).

4. Given the following dynamics for system representation

$$\dot{x}_1(t) = x_2(t) - 3x_1(t) - u(t)$$
$$\dot{x}_2(t) = x_1(t)$$

and we directly measure the state $x_2(t)$.

a) Find A, B, C, D for the above description. Is the system model internally stable? (**1 point**).

$$\dot{x}(t) = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

$eig(A) \Rightarrow \lambda_1 = -3.3028, \lambda_2 = 0.3028$ Not stable, there is a positive real eigenvalue

b) By using the coefficient matrices (A, B, C, D), find the transfer function $G(s)$ (**1 point**).
$G(s) = C(sI - A)^{-1}B = \frac{-1}{s^2+3s-1}$

c) Based on $G(s)$, is this a non-minimum phase representation? Is the system model $G(s)$ input-output stable? (**1 point**)!
There is no zero-pole cancellation in the transfer fucntion. It is NMP system model, we have no zeros at RHP (we have none actually). It is IO unstable, the poles of the transfer fucntion is the same as the eigenvalues. Note, internally unstable model can be seen as stable from IO perspective!

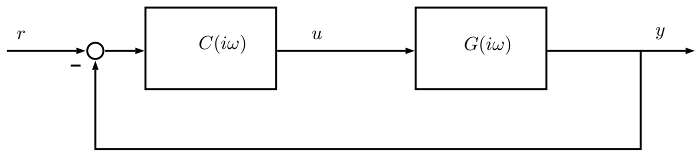
d) What is the steady-state value for $y_{(t)}$ while $t \mapsto \infty$ if $u(t) = 1 \; \forall t > 0$ is applied? (**1 point**)
Since the system model is unstable, FVT can not be directly used. Due to the unstable nature of the model however, $y(\infty)$ goes ∞, no steady state exists.

e) Is the system model controllable and observable? (**1 point**).
Yes, the rank conditions have to be calculated to justify them, the reachability and observability matrices are of full rank.

5. Given a PI controller $C(i\omega) = K_p(1 + \frac{1}{i\omega})$ and a transfer function for a model described by $G(i\omega) = \frac{6i\omega}{(i\omega+1)((i\omega)^2+3i\omega+2)}$ in a closed loop setup depicted in Figure 3.

   a) Find K_p such that $\varphi_m = 40$ degrees. (**2 point**).

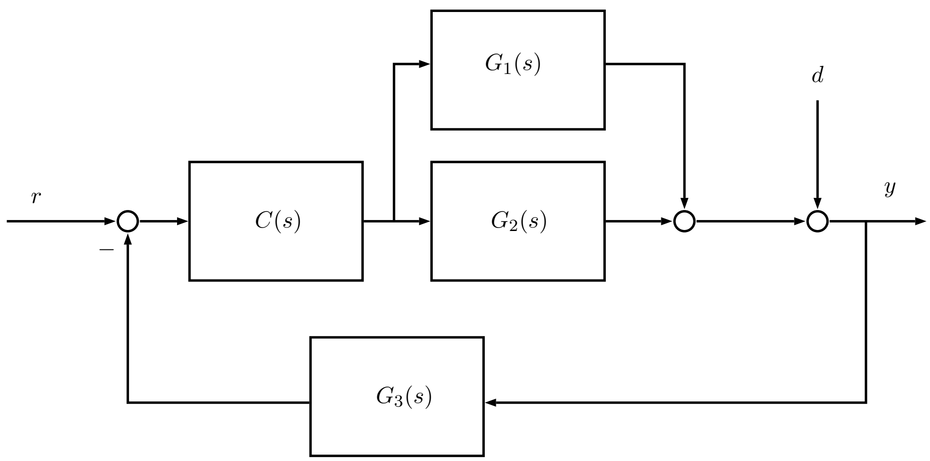
   b) What is the gain margin with K_p found in a)? (**1 point**)



Figur 3: Återkopplat blockschema

$K_p \approx 3$, $g_m \approx 11dB$

6. Given the closed-loop system as shown in Figure 4



Figur 4: Closed-loop block diagram

$$G_1(s) = \frac{s(s+3)}{(s+2)(s+1)},\ \ C(s) = s + 3 + \frac{2}{s},\ \ G_3(s) = \frac{1}{5},\ \ G_2(s) = \frac{s}{s+3}$$

   a) Find the sensitivity function from the above closed loop setup (i.e the transfer function from the signal d to the output y one, while $r = 0$) (**1 point**).

$$S(s) = \frac{1}{1 + C(s)(G_1(s) + G_2(s))G_3(s)}$$

$$C(s)(G_1(s) + G_2(s))G_3(s) = \frac{1}{s}(s^2 + 3s + 2) \cdot \left(\frac{s(s+3)}{(s+2)(s+1)} + \frac{s}{s+3}\right) \cdot \frac{1}{5} =$$

$$\frac{1}{s}((s+2)(s+1)) \cdot \left(s\frac{(s+3)^2 + (s+2)(s+1)}{(s+2)(s+1)(s+3)}\right)\frac{1}{5} = \frac{1}{5}\left(\frac{(s+3)^2 + (s+2)(s+1)}{(s+3)}\right)$$

$$S(s) = \frac{1}{1 + C(s)(G_1(s) + G_2(s))G_3(s)} = \frac{s+3}{s^2 + 7s + 13}$$

   Stable poles for $S(s)$.

   b) What is the value of $\int_0^\infty \log |S(i\omega)| d\omega$ (hint: Bode sensitivity criteria) (**1 point**).
   Due to the fact $S(s)$ has stable poles, the Bode integral gets 0.

7. Given the following state-space representation and cost functional by,

$$\dot{x}(t) = 0.5x(t) + u(t)$$
$$J(u) = \frac{1}{2}\int_0^\infty \left(x^T(\tau)x(\tau) + u^T(\tau)Q_u u(\tau)\right) d\tau$$

After minimizing the cost function (LQR), the closed-loop system's eigenvalue is located at -1. Find Q_u and $\bar{P}$ (**2 point**).

$$A = 0.5, B = 1, Q_x = 1$$
$$Q_u =?,\ \bar{P} =?$$
$$A - B\bar{K} = -1 = 0.5 - Q_u^{-1}\bar{P} \Rightarrow \bar{P} = 1.5Q_u$$
$$\bar{P}A + A^T\bar{P} - \bar{P}BQ_u^{-1}B^T\bar{P} + Q_x = 0$$
$$\bar{P} + 1 - \bar{P}\frac{1.5}{\bar{P}}\bar{P} = 0 \Rightarrow \bar{P} = 2,\ \Rightarrow Q_u = \frac{4}{3}$$