Chalmers Tekniska Högskola
Institutionen för signaler och system
Avdelningen för reglerteknik

Tentamen i Reglerteknik SSY310/ERE091

Unique exam identification number: 0165392
English version

1. Time: 4 hours. You need to be remotely supervised and ID-checked via Zoom. You need to have a computer/device with camera and microphone.
2. Examiner: Balazs Kulcsar (☎1785, ZOOM, Skype for Business)
3. 20 points can be reached in total (resolution 0.5 points). Table **??** shows the limits for each grade.

Tabell 1: Grades

Points	Grade
≤ 9.5	Failed
$10 \ldots 12.5$	3
$13 \ldots 15.5$	4
$16 \ldots 20$	5

4. Random sets of exam questions may randomly be assigned to students.
5. Handwritten solutions are requested (name and cid on each pages). When done, scan/photograph your solutions and compile it into one pdf document. Upload your file to Canvas (submission site closes 30 min after the examination). If the electronic version of the solution is not readable from the file, it will not be assessed (with 0 point).
6. Cooperation with or external help from other person is prohibited during examination! If signs of cooperations/external help are being discovered, *all exams involved will automatically be disqualified* and we will *automatically* report the *case to Chalmers with a request for suspension at Swedish Higher Education Authority.*
7. We may call students for oral post-check of solutions in the exam period (within a few days after the exam). Then, students will be asked (with short notice) to explain their solutions or answers.
8. All other aids can be used (books, notes, Matlab, etc.).
9. Teacher(s) will online be available. Examination results will be advertised approx 10 days after the exam. Inspection of results via Zoom.

Good luck!

Questions

1. Briefly answer the questions below.

   a) True or false (briefly motivate your answer)? Given a PID controller $C(s) = K_p(1 + T_d s + \frac{1}{T_i s})$. The transfer function $C(s)$ is strictly proper with relative degree 1. (**1 point**)

   $C(s) = \frac{K_p s + K_p T_d s^2 + K + p/T_i}{s}$ and hence $deg(b(s)) = 2 > deg(a(s)) = 1$. This is an improper transfer function. There is no relative degree equal to 1, relative degree is a non-negative number and gets defined for proper transfer functions.

   b) Show that at disturbance feedforward compensation ($F(s)$-feedforward term, $C(s)$- feedback gain), the $F(s)$ never influences the closed loop stability. (**1 point**)

   Draw the block diagram for feedforward compensation, the disturbance is measured. Use $F(s) \approx G_d(s)$ so that the effect of the disturbance is cancelled. Since the closed loop transfer function for $\frac{Y(s)}{R(s)} = \frac{G(s)C(s)}{1+G(s)C(s)}$, the $F(s)$ does not influence stability.

   c) True or false (briefly motivate your answer)? Non-minimum phase system models always have unstable (RHP) zeros and stable poles (LHP). (**1 point**)

   False. Class of NMP models is much larger than models with RHP zeros only. Counter example: delayed modes.

   d) Assume the small gain theorem is fulfilled for a closed loop system model shown in Figure **??**. What can you then conclude? (**1 point**)

   $\forall \omega \ |C(i\omega)||G(i\omega)| < 1$ means that the closed loop is stable. Amplifications in the loop are such that they never gets larger than 1.

   e) Given a state estimator based state feedback controller. Is that a dynamic or static output feedback controller? Briefly motivate your answer. (**1 point**)

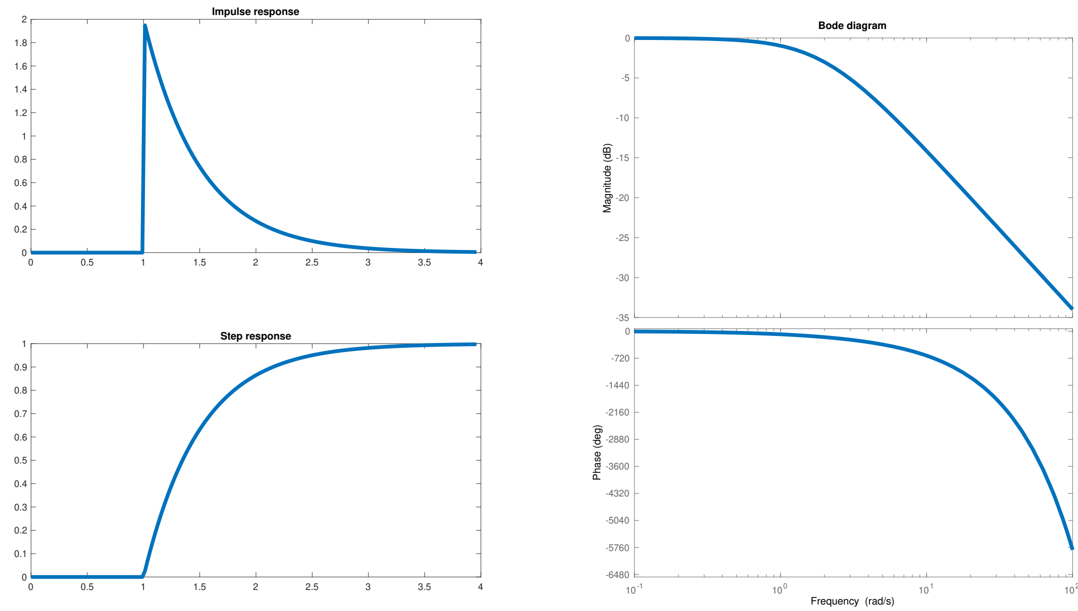
   State estimation is a dynamic process and it makes the observer based controller design dynamic. Since observers only use measured input and output variables, it is a dynamic output feedback control policy

2. Figure **??** depicts the impulse, step and bode plots for a transfer function $G(s) = G_1(s) \cdot \frac{A}{Ts+1}$ (the system model and the all inputs are in restfor $t < 0$).

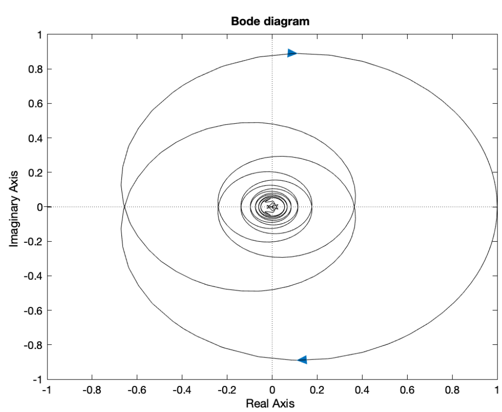
   a) Find $G_1(s)$, A and T based on the plots.(**1 point**)

   b) Sketch the Nyquist plot of $G(s)$ (approximate form).(**0.5 point**)

   $A = 1, T = 0.5$ from IVT, FVT and $G_1(s) = e^{-s\tau}$ with $\tau = 1$, first order lag with delay see plot in Fig **??**.



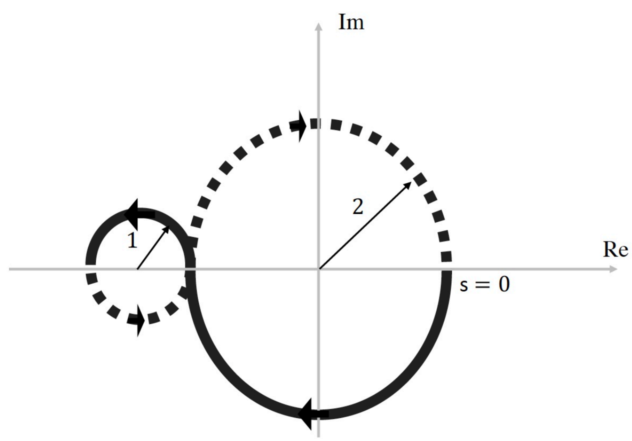
Figur 1: Plots for $G(s)$



Figur 2: Nyquist plot

3. Given the Nyquist plot of a loop transfer function $L(s) = C(s)G(s) = K_pG(s)$, see Figure **??**. $L(s)$ has no open loop unstable poles.

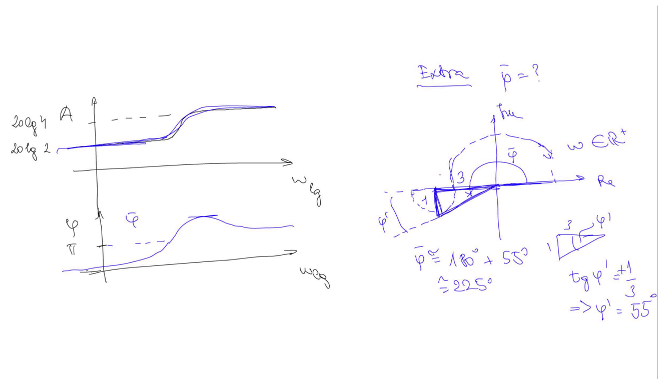
   a) Is the closed loop stable (under unity feedback, see Figure **??**) (brief motivation is needed)?(**1 point**)
   b) Find K_p such that the gain margin will be 2 (**1 point**)
   c) What is the phase margin associated to the answer found in *b*)? (**0.5 point**)
   d) Sketch the Bode diagram of the model described by the plot in Figure **??**. (**0.5 point**)



Figur 3: Nyquist plot of $L(s)$ with $K_p = 1$. Solid and dashed line plots are semi-circles with radii indicated on the symmetric plot.

a) Unstable, it encircles -1.
b) $K_p = \frac{1}{2 \cdot 4}$ radially shrinks $L(s)$ and makes the gain margin 2.
c) ∞, because at $K_p = \frac{1}{8}$ even if you infinitely many times rotate the the loop transfer function, there is no hit of $L(s)$ at -1 point.

d)



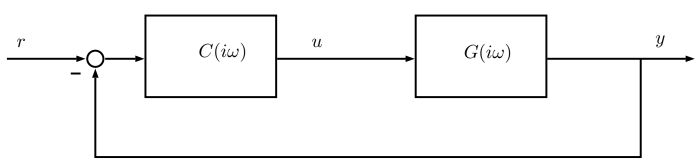
4. Given a PD controller $C(i\omega) = K_p(1 + K_d i\omega)$ and a transfer function for a pandemic model described as $G(i\omega) = \frac{4}{(i\omega+2)(i\omega+3)(i\omega+4)}$ in a closed loop setup depicted in Figure **??**.

   a) Someone set a value for K_p and let the controller run in closed loop. Given we know $K_d = 0.25$ and the closed loop tracking error $\frac{r_\infty - y_\infty}{r_\infty} = 1 - 0.625$ for $r_\infty = 1$ (step input). What is the value for K_p?(**1 point**).

   b) Can you find the phase margin associated to the controller parameter K_p found in a) and $K_d = 0.25$? (**1 point**).

   c) Redesign K_p. Find K_p ($K_d = 0.25$) such that the phase margin is $\frac{\pi}{4}$ (**1 point**)

   d) What is the steady state error ($\frac{r_\infty - y_\infty}{r_\infty}$) for K_p ($K_d = 0.25$) found in c)? (**1 point**)



Figur 4: Återkopplat blockschema

a) The loop transfer function gets simplified to $L(i\omega) = \frac{K_p}{(i\omega+2)(i\omega+3}$ and for the closed loop when using FVT we get $\frac{K_p}{K_p+6} = 0.625 \Rightarrow K_p = 10$

b) Since $\nexists \omega \, |L(i\omega)| = 1$ the phase margin gets ∞

c) $K_p \approx 80$

d) $\frac{K_p}{K_p+6} = \frac{80}{86} \approx 0.91 \Rightarrow 9\%$ tracking error.

5. Given the following transfer function,

$$G(s) = \frac{s+\gamma}{(s+\alpha)(s+\beta)}$$

with real valued scalars $-\infty < \alpha, \beta < 0$, and $0 < \gamma < \infty$.

a) Assuming controllability is always satisfied by α, β, γ, find the controllability canonical state space representation. (**0.5 point**)

b) Is the controllability canonical state space representation stable? (**1 point**)

c) Find the state feedback ($u(t) = -Lx(t) + L_r r(t)$) gain $L = [\ell_1 \ \ \ell_2]$ and L_r. First find L that moves the closed loop eigenvalues to -1 and -2 for any the values α, β (L may depend on α, β). Second, using L find L_r that ensures asymptotic tracking of $r(t)$ by $y(t)$ for any values of γ (L_r may depend on γ). (**2 point**)

d) Is *a*) observable for all values of α, β, γ? (**1 point**)

e) Find a diagonal state space realization of $G(s)$ (**1 point**)

f) Using *e*) and setting $\alpha = -1, \beta = -2$ and $\gamma = 3$ find $y(1)$ if $u(t) = 0$ and $x_0 = [1 \ \ 1]^T$ (i.e. the value of the output at $t = 1$ given the initial condition and the zero input value) (**1 point**)

a)

$$A = \begin{bmatrix} -\alpha-\beta & -\alpha\beta \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \gamma \end{bmatrix}$$

b) $det(I\lambda - A) = 0 \Rightarrow \lambda_1 = -\alpha$ and $\lambda_2 = -\beta$, and since $-\infty < \alpha, \beta < 0$ those are unstable real eigenvalues.

c) $L = [3 - (\alpha+\beta) \ \ 2 - \alpha\beta]$, $L_r = -2\gamma^{-1}$

d) $\mathcal{O} = \begin{bmatrix} 1 & \gamma \\ \gamma-\alpha-\beta & -\alpha\beta \end{bmatrix}$ and $det\mathcal{O} = -\alpha\beta - \gamma^2 + \gamma(\alpha+\beta) = 0 \Rightarrow$ quadratic relation for γ. All coefficients are negative, so based on e.g. Viete formulas, roots are having negative sings too. In other way, $\gamma_{1,2} = \frac{-\alpha-\beta \pm \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}{-2} \Rightarrow 0 < -\alpha\beta$. No proper root exists for γ. It is observable under the constraints and conditions given.

d) $\frac{K_p}{K_p+6} = \frac{80}{86} \approx 0.91$

e) $G(s) = \frac{r_1}{s+\alpha} + \frac{r_2}{s+\beta}$ with $r_1 = \frac{\gamma-\alpha}{\beta-\alpha}$, $r_2 = \frac{\gamma-\beta}{\alpha-\beta}$ + the diagonal state space with $\lambda_1 = -\alpha$ and $\lambda_2 = -\beta$

f) The diagional form gets $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -4 \\ 5 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix} \Rightarrow y(1) = Ce^{A \cdot 1}x_0 \approx 10.1$