Chalmers Tekniska Högskola
Institutionen för signaler och system
Avdelningen för reglerteknik

Tentamen i Reglerteknik SSY310/ERE091

August 30th, 2019, English version

1. Time: 4 hours.
2. Examiner: Balazs Kulcsar (☎1785)
3. 20 points can be reached in total (resolution 0.5 points). Table 1 shows the limits for each grade.

Tabell 1: Grades

Points	Grade
≤ 9.5	Failed
$10 \ldots 12.5$	3
$13 \ldots 15.5$	4
$16 \ldots 20$	5

4. The following aids are allowed:
   - Self-hand written formula sheet"**A4 format, single page with hand written notes on ONE side (no copies are allowed).**
   - **Pocket calculator (non-programmable, without graphical plotting function, cleared memory before starting, including Casio fx-991 types).**
   - **Beta, Physics handbook.**

   Prohibited items: other books, lecture notes, phone, tablet, computer or similar.
5. Teacher will visit and answer questions during the first and last hours of the allotted exam time (held at Chalmers).

Good luck!

Questions

1. Briefly answer the questions below.

   a) Define linearity and time invariance for dynamic system models. (**1 point**)
   Linearity means that the principle of signals superposition holds true. Time invariant system models are shift invariant ones by τ time delay they give the same outputs.

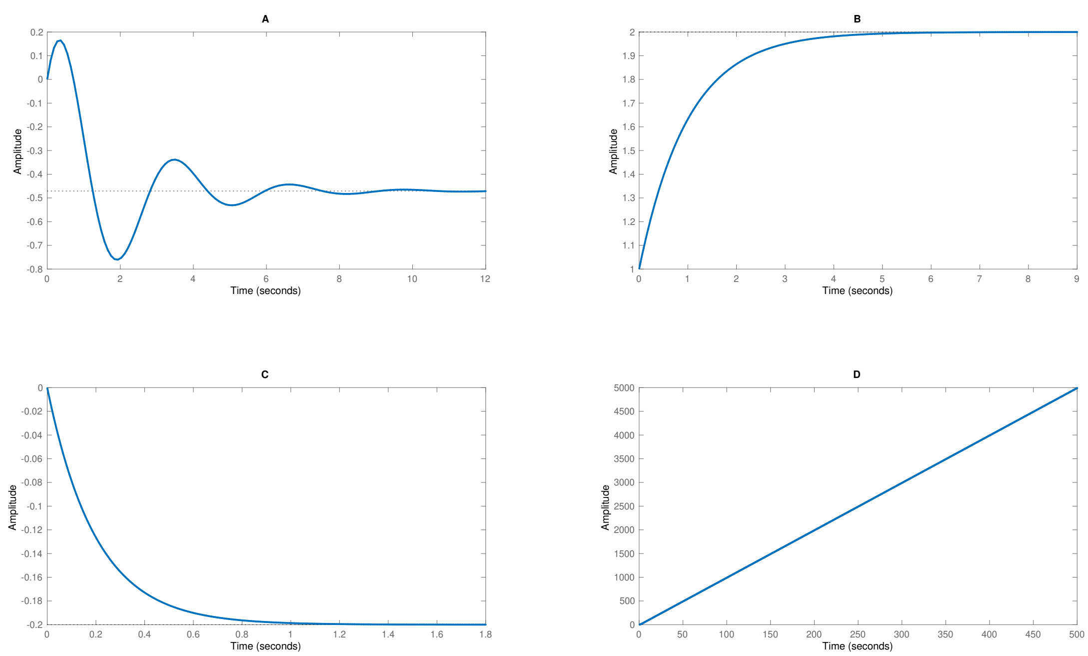
   b) True or false (briefly motivate your answer)? All controllable state space representations are internally stable. (**1 point**)
   False, the state space representation can be controllable (Kalman rank condition) but unstable (eigenvalues with positive real parts).

   c) Define the Routh matrix. Why is it useful? (**1 point**)
   Definition of Routh matrix comes here. Under some condition, it can be used to conclude input output stability of a transfer function.

   d) Explain the concept of eigenvalue assignment in state observer design. (**1 point**)
   Given a pre-defined set of (stable) closed-loop eigenvalues, find the observer gain that moves open loop eigenvalues to the predefined ones. The key is to use the difference of coefficients between the open and the closed loop characteristics equations in observer canonical form.

2. Match (with brief explanation) the step response of following state space model to either of the plots in Fig. 1 (**2 point**),

$$
\begin{aligned}
\dot{x}(t) &= u(t) - x(t)\\
y(t) &= x(t) + u(t),\\
x(0) &= 0, x, u, y \in \mathbb{R}.
\end{aligned}
$$

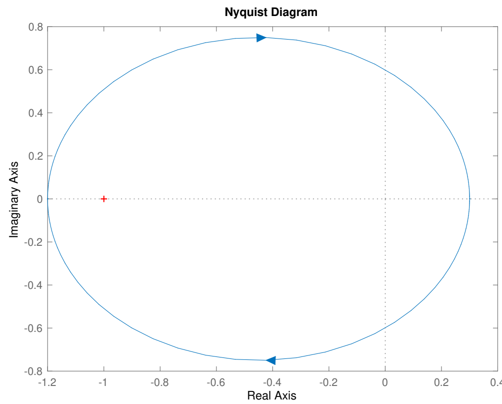


Figur 1: Step responses

B, Laplace transform first resulting in a first order TF with direct feedthrough, Initial Value Theorem.

3. Given the proportional controller $C(s) = K_p = 0.3$ and the transfer function $G(s) = 1 - \frac{10}{s+2}$. Based on Nyquist plot (**1 point**), conclude closed-loop stability (**1 point**). For the above system, find (if exists) all stabilizing P controller ($0 < K_p < \infty$) (**1 point**).

See plot and conclusion in Fig. 2 Since, $L(s) = \frac{K_p s - 8K_p}{s+2}$, K_p moves the Nyquist plot along the Re axis.

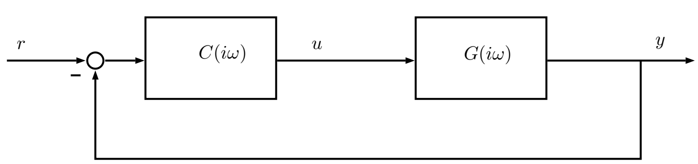


Figur 2: Nyquist plot, $L(s)$ encircling -1 hence the closed loop is unstable

Increasing a finite positive K_p will shift $L(s)$ to the left, encircling -1. Decreasing K_p will move $L(s)$ to the right. Note $\lim_{s \mapsto 0} L(s) = -K_p/4 < 1$ has to be such that it does not take the value -1, so $K_p < 1/4$

4. Given an P *controller*, and $G(i\omega) = \frac{1}{(5i\omega+1)(10i\omega+1)}$ see in Figure 3.

   a) Find the free parameter in the controller to ensure $\varphi_m = 45°$. (**2 point**)
   $K_p \approx 7.6$.
   b) How large is the steady state tracking error $|y(\infty) - r(\infty)|/|r(\infty)|$? (**1 point**)
   $|\frac{7.6}{1+7.6} - 1| \approx 0.116 \Rightarrow 11.6\%$.
   c) What is the gain margin associate to a)? (**1 point**)
   ∞ large, reaching $-\pi$ at $\omega = \infty$ where the gain is $-\infty$



Figur 3: Återkopplat blockschema

5. Given the following state-space representation by,

$$\dot{x}(t) = \begin{bmatrix} \alpha & -4 \\ 1 & 4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 2 & 0 \end{bmatrix} x(t) + \beta u(t)$$

with finite scalars $0 < |\alpha|, |\beta| < \infty$, where $u(t), x(t), y(t)$ are control input, state vector and measured output, respectively.

a) Is the state-space representation asymptotically stable for all values of α? (**1 point**)

Not,

$$det(\lambda I_2 - A) = \lambda^2 - (\alpha + 4)\lambda + 4(1 + \alpha) = 0, \;\; \lambda_{1,2} = \frac{(\alpha + 4) \pm \sqrt{\alpha^2 - 8\alpha}}{2}$$

e.g. $\alpha \geq 0$ we get unstable eigenvalues, it is enough to show with one value to answer the question.

b) Is the state-space representation of minimal order for all α? (**2 point**)

Not, checking the reachability we found the state space not reachable with $\alpha = 9$. The determinant condition for observability is independent of α.

c) With $\beta = \alpha = -4$ design a state feedback gain by eigenvalue assignment that allocates the closed loop eigenvalues to $\bar{p}_1 = -1$ and $\bar{p}_1 = -2$. (**2 point**)

Controllability canonical form is needed. Therefore, first T has to be found (1 point) and then by following the definition of eigenvalue assignment $L = \begin{bmatrix} -0.0769 & 2.9231 \end{bmatrix}$ (1 point)

d) With $\beta = \alpha = -4$ design a state observer gain by eigenvalue assignment that allocates the observers' eigenvalues to $\bar{p}_1 = -1$ and $\bar{p}_1 = -2$. (**2 point**)

Observable canonical form is needed. Therefore, first T has to be found (1 point) and then by following the definition of eigenvalue assignment $K = \begin{bmatrix} 1.5 \\ -3.25 \end{bmatrix}$ (1 point)