Chalmers University of Technology
Department of Signals and Systems
Automatic Control Group

Resit exam questions for Reglerteknik SSY310/ERE091

August 27th, 2015

Cover page

1. Timeframe: 4 hours.

2. Examiner: Balazs Kulcsar, internal phone number: 1785, `kulcsar@chalmers.se`

3. Necessary condition to obtain the exam grade is *i*) for SSY310 to have approved assignments 1-2, project work and labs 1-2, *ii*) for ERE091 to have approved labs 1-2.

4. 20 points can be reached in total (with 0.5 point resolution), Table 1 shows the grading system.

Table 1: Grading system

Points	Grade
≤ 9.5	fail
$10 \ldots 12.5$	3
$13 \ldots 15.5$	4
$16 \ldots 20$	5

5. You *can* use the following aids:

   - Self-hand written "formula sheet" **A4 format, single pager** with hand written notes on ONE side (no-copies are allowed).
   - Pocket calculator (non-programmable, without graphical plotting function, cleared memory before starting, including Casio fx-991 types).
   - Mathematical and physics handbook (Beta, Physics).

   Prohibited to use: other books, lecture notes, phones, tablets, computers, any other communication devices.

6. Teacher(s) will show up in person during the examination (in the first and last hour). Examination results will be advertised on September 7th 2015 (pingpong.chalmers.se). Inspection of results on September 7th 10-11 am, E-building floor 5, room 5435A.

Good luck!

Questions

1. Answer the following questions in a few sentences each.

   a) Explain briefly the notion of Nyquist stability criteria. (**1 point**)

   b) What does it mean similarity state transformation? Explain it. Does it influence the transfer function? (**1 point**)

   c) Explain the notion non-minimum phase system. Give an example to it. (**1 point**)

2. Match and explain!

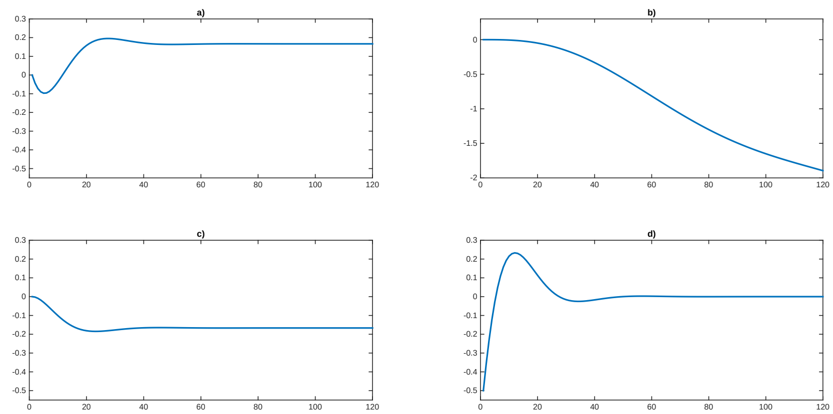

Figure 1: Step and impulse responses

Depicted in Figure 1 are 2 step response functions and 2 impulse response functions. Two of them belong to the same dynamical system. Match an impulse and a step response to each of the transfer functions (briefly motivate your choice!). $G_1(s) = \frac{0.5(1-s)}{s^2+2s+3}$, $G_2(s) = \frac{0.05}{s(-0.1s^2-0.2s-0.3)}$. (**2 point**)

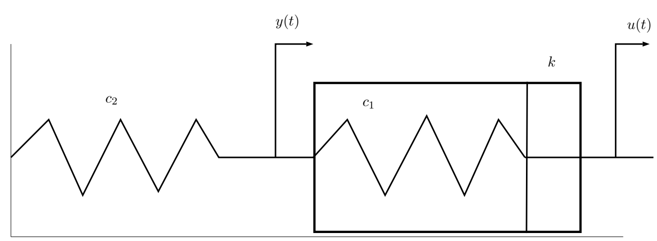

Figure 2: Spring-damper mechanical system

3. Given a mechanical system description as shown in Figure 2 with displacements $u(t), y(t)$ in $[m]$. With spring coefficients $c_1 = 1N/m; c_2 = 4N/m; anddampingcoefficientk = 20Ns/m$

   (a) Find the frequency function of the system. (**1 point**)

   (b) Plot the Nyquist diagram of the previously found frequency function (**1 point**)

4. In Figure 3 is given a closed-loop block-diagram.

   (a) Find the complementary sensitivity function of the closed-loop system. (**1 point**)

   (b) Plot the connected Bode diagram. (**1 point**)

   (c) Compute the value for F such that $r_\infty = y_\infty$. (**1 point**)

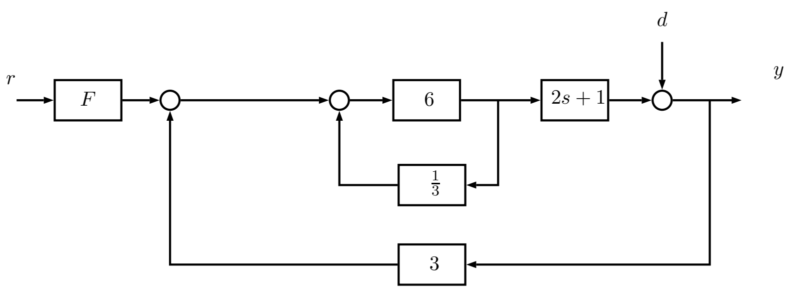

Figure 3: Blockdiagram

5. Given a proportionel type controller $C(i\omega) = K$ and the plant's frequency function $G(i\omega) = \frac{6}{5(i\omega)^3+8(i\omega)^2+3(i\omega)}$ (with unity feedback, see Figure 4). Find K such that the closed-loop has $\varphi_m = 45°$ (Use the lin-log paper attached).(**2 point**)

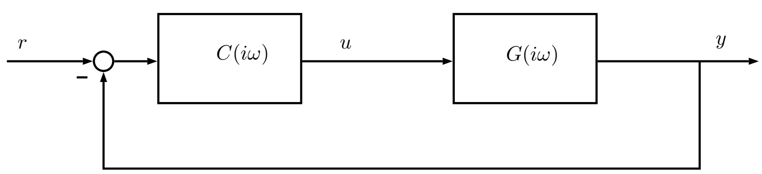

Figure 4: Closed-loop setup

6. By means of the small gain theorem, find the finite value of a proportional controller $K > 0$ that robustly stabilizes the nominal transfer function $G_n(s) = \frac{1}{s+1}$ if the upper bound of the additive uncertainty $d_a = \frac{s}{0.33(0.33s+1)}$ (unity feedback applies). (**2 point**)

7. Given the following state space representation by,

$$\dot{x}(t) = \begin{bmatrix} -1.5 & -\beta \\ -1 & \gamma \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + \sqrt{\beta \cdot \gamma}\, u(t)$$

   where β and γ are real valued finite scalars.

   a) Find value(s) of β and γ for which the system becomes minimal order (**2 point**).

   b) With $\beta = \gamma = 1$, is the system stable? (**1 point**)

   c) With $\beta = \gamma = 1$, find the transfer function of the system. (**1 point**).

8. (*)Consider the following continuous time system given by a state space representation as,

$$\dot{x}_1(t) = x_2(t) + u(t)$$
$$\dot{x}_2(t) = u(t)$$
$$J(u) = \frac{1}{2}\int_0^\infty \left(x^T(t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + u^2(t) \right) dt.$$

Find the optimal (steady-state LQR) feedback gain that minimizes $J(u)$ by applying the diagonal solution structure as $\bar{P} = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix}$ (**2 point**).