Anonymous code: ______________________________

	1	2	3	4	5	6	7	8	Grade
Results:									

Basic question 1: Insertion sort

Assume you want to sort each of the following arrays using insertion sort:

```
A. [4, 7, 2, 2, 5, 4]
B. [2, 2, 4, 4, 5, 5]
C. [1, 7, 2, 6, 3, 4]
D. [7, 6, 5, 4, 2, 1]
E. [6, 3, 5, 3, 6, 1]
```

Which array is the worst (uses the most number of array accesses),
and which is the best (uses the least)?

Worst: ______________________

Best: ______________________

Brief explanation:

Basic question 2: Linearising a BST into a list

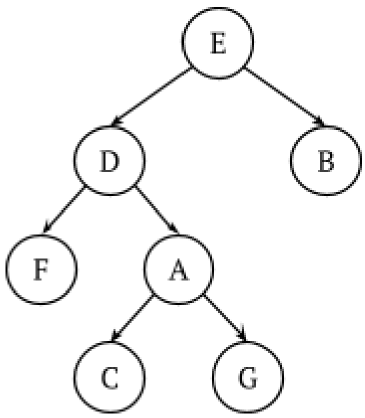
Assume the following code that transforms a binary search tree into a list of its values:

```
def f(bst) -> list:
    result = empty list
    agenda = empty stack or queue
    add the root of bst to agenda

    while agenda is not empty:
        node = remove from agenda
        if node is not null/None:
            add node.left to agenda
            append node.value to the end of result
            add node.right to agenda

    return result
```

Now assume you give it the BST to the right as input:



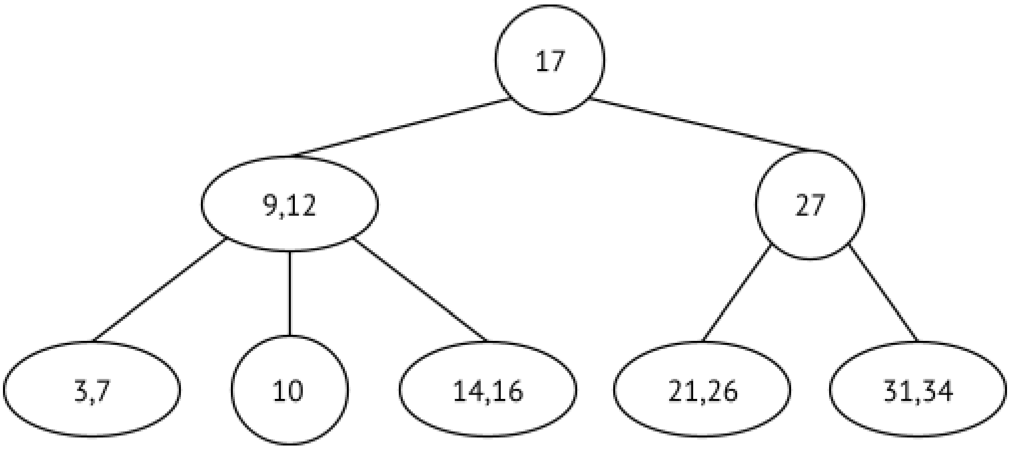
In what order will the values be returned…

(A) …if we use a ***stack*** as the agenda?

(B) …if we use a ***queue*** as the agenda?

Basic question 3: Insertion in 2-3 trees

Given the following 2-3 tree:



Insert first 32 and then 25 into this 2-3 tree. Show how the tree looks after each insertion.

After inserting 32:

After inserting 25:

Basic question 4: Priority queues

Assume the following binary min-heap:

0	1	2	3	4	5	6	7	8	9	10	11
2	7	5	10	13	8	16	16	13	15		

Insert the number 6 into the heap and show the result here:

0	1	2	3	4	5	6	7	8	9	10	11

Delete the minimum element from the resulting heap and show the result here:

0	1	2	3	4	5	6	7	8	9	10	11

Optionally you can also draw the heaps as trees here:

Basic question 5: Hash tables

Here is an open addressing hash table of strings, which uses standard modular hashing and +1 linear probing:

0	1	2	3	4	5	6	7	8	9
H	A	V	E				F	U	N

The hash codes for each of the strings are the following:

$$h(\mathrm{A}) = 38,\ h(\mathrm{E}) = 20,\ h(\mathrm{F}) = 97,\ h(\mathrm{H}) = 49,\ h(\mathrm{N}) = 27,\ h(\mathrm{U}) = 57,\ h(\mathrm{V}) = 42$$

In which order can the elements have been inserted into the hash table?

Check the box(es) ☑ with the correct alternative(s) – there can be more than one:

- ☐ V F U N H A E
- ☐ H A V E F U N
- ☐ F U N H A V E
- ☐ F V U N H E A
- ☐ F V N U H A E

Now, pick one of the correct alternatives and insert the elements in ***reversed order*** into the following linear probing hash table:

0	1	2	3	4	5	6	7	8	9

In which order did you insert the elements? ____________________________

Basic question 6: Quicksort partitioning

Perform a quicksort partitioning in-place of the following array:

0	1	2	3	4	5	6	7	8
17	97	21	34	37	1	13	42	33

Use the element at index 3 (i.e., the number 34) as pivot, and then partition. Recall that the partitioning algorithm also swaps the pivot into the correct place afterwards.

Note: quicksorting the left and right parts of the partition is not part of this question.

Write down how the array looks after the partitioning:

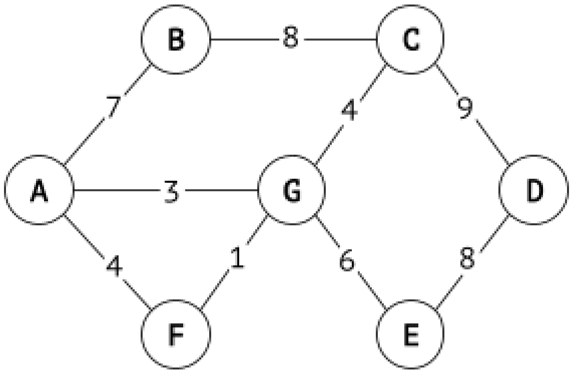
0	1	2	3	4	5	6	7	8

What sequence of swaps did you make when partitioning the array?

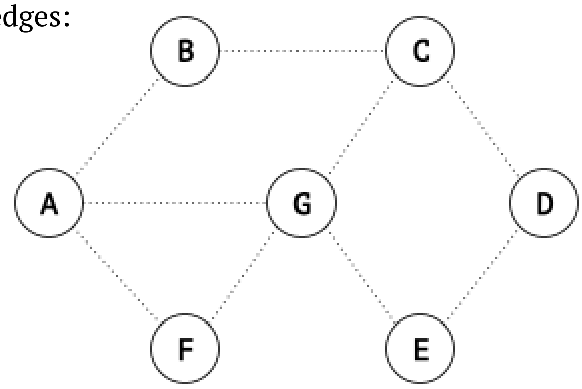
If you used a different algorithm from that of the course, explain it here:

Basic question 7: Graphs

Here is a weighted undirected graph:



Perform **Prim's algorithm** to construct the minimum spanning tree (MST) for the graph.
Draw the MST by filling the dotted edges:



List the edges of the MST *in the order they are produced* by the algorithm.
Do this for two different starting vertices.

- Write the edges in the form AC, DF, …
- You do not have to use all rows.

Starting vertex: A

1	
2	
3	
4	
5	
6	
7	
8	

Starting vertex: D

1	
2	
3	
4	
5	
6	
7	
8	

Basic question 8: Complexity

Here is a sorting algorithm that uses a minimum priority queue for sorting an array of numbers.

```
def sort(array):
    S = new empty binary heap

    for each x in array:
        insert x into S

    i = 0
    while S is not empty:
        y = delete minimum from S
        array[i] = y
        i += 1
```

What is the worst-case asymptotic time complexity in the size *n* of `array`? Answer for each of the two loops, and the total complexity. Write your answer in O-notation, and be as exact and simple as possible.

Complexity of the for loop: O(________________________________)

Complexity of the while loop: O(________________________________)

Complexity of the function sort: O(________________________________)

Brief explanation:
(Alternatively, you can add comments directly to the code above if you want.)

Anonymous code: ____________________

	1	2	3	4	5	6	7	8	Grade
Results:									

Basic question 1: Insertion sort

Assume you want to sort each of the following arrays using insertion sort:

```
A. [4, 7, 2, 2, 5, 4]
B. [2, 2, 4, 4, 5, 5]
C. [1, 7, 2, 6, 3, 4]
D. [7, 6, 5, 4, 2, 1]
E. [6, 3, 5, 3, 6, 1]
```

Which array is the worst (uses the most number of array accesses),
and which is the best (uses the least)?

Worst: D (reversely sorted)

Best: B (already sorted)

Brief explanation:

If the list is already sorted, then the inner loop will exit immediately because it finds that the element is already in its right place.

If the list is reversely sorted, then it has to move every element as far as possible to put it in its right place.

Basic question 2: Linearising a BST into a list

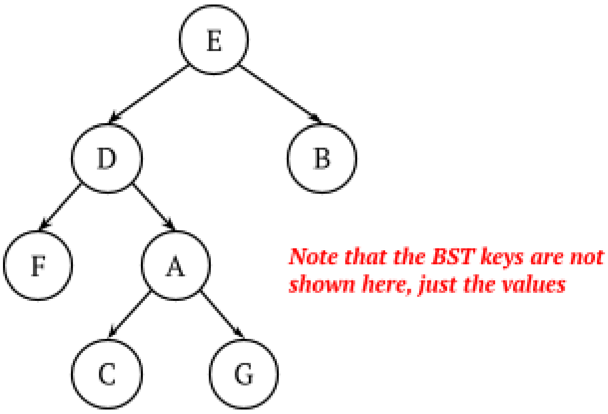
Assume the following code that transforms a binary search tree into a list of its values:

```
def f(bst) -> list:
    result = empty list
    agenda = empty stack or queue
    add the root of bst to agenda

    while agenda is not empty:
        node = remove from agenda
        if node is not null/None:
            add node.left to agenda
            append node.value to the end of result
            add node.right to agenda

    return result
```

Now assume you give it the BST to the right as input:



In what order will the values be returned...

(A) ...if we use a ***stack*** as the agenda?

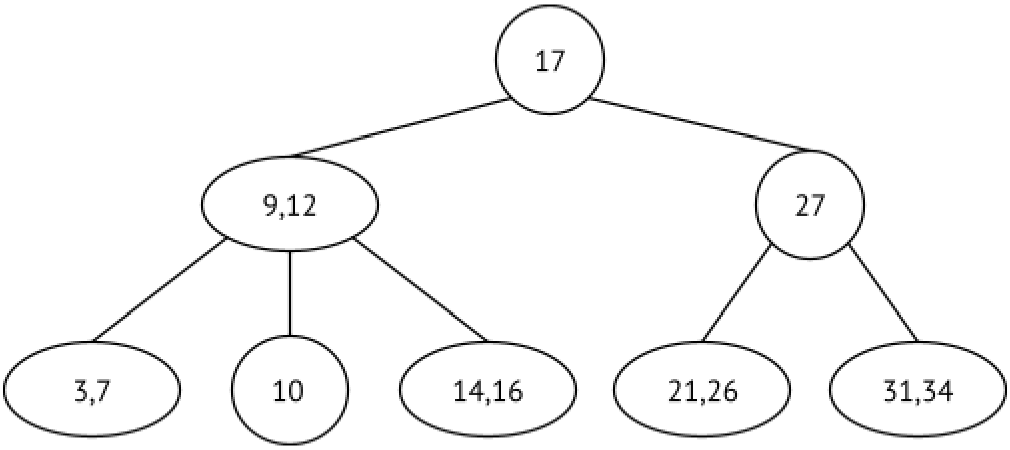
E	B	D	A	G	C	F

(B) ...if we use a ***queue*** as the agenda?

E	D	B	F	A	C	G

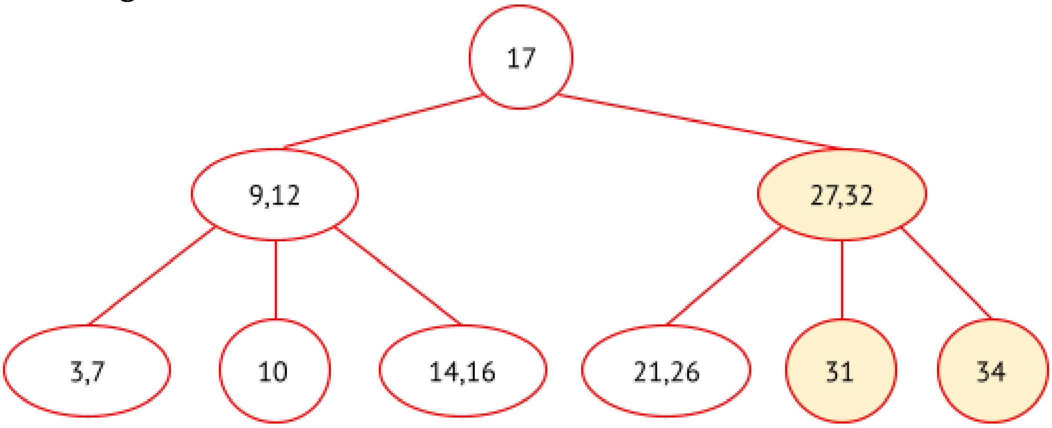
Basic question 3: Insertion in 2-3 trees

Given the following 2-3 tree:

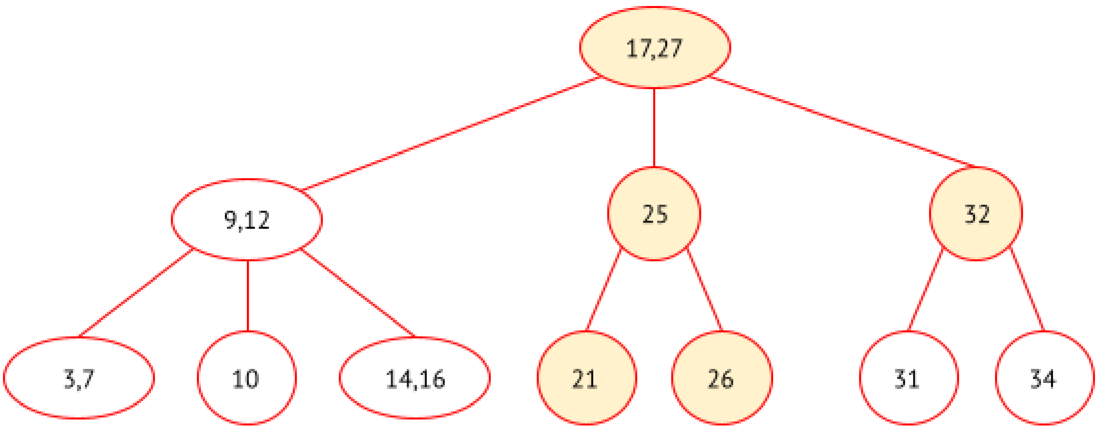


Insert first 32 and then 25 into this 2-3 tree. Show how the tree looks after each insertion.

After inserting 32:



After inserting 25:



(the nodes with yellow background are the ones that change)

Basic question 4: Priority queues

Assume the following binary min-heap:

0	1	2	3	4	5	6	7	8	9	10	11
2	7	5	10	13	8	16	16	13	15		

Insert the number 6 into the heap and show the result here:

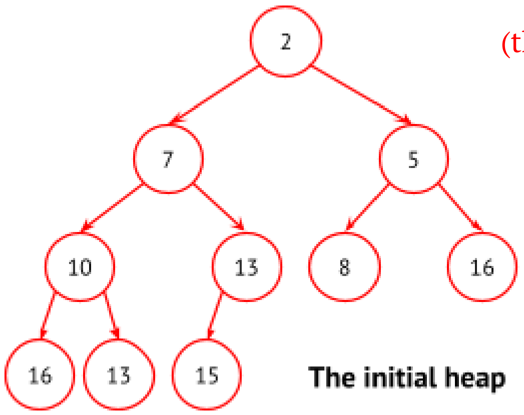
0	1	2	3	4	5	6	7	8	9	10	11
2	6	5	10	7	8	16	16	13	15	13	

Delete the minimum element from the resulting heap and show the result here:

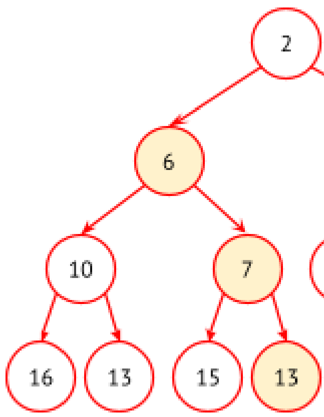
0	1	2	3	4	5	6	7	8	9	10	11
5	6	8	10	7	13	16	16	13	15		

Optionally you can also draw the heaps as trees here:

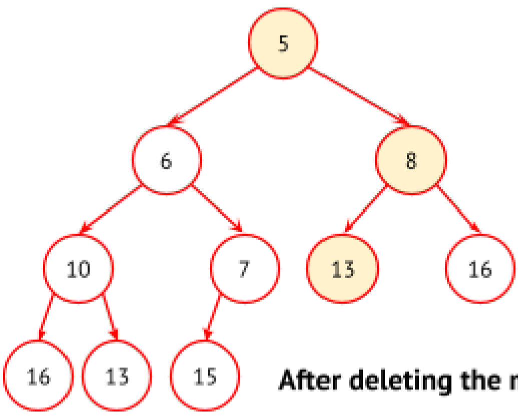
(the cells with yellow background are the ones that change)



The initial heap



After inserting 6



After deleting the minimum

Basic question 5: Hash tables

Here is an open addressing hash table of strings, which uses standard modular hashing and +1 linear probing:

0	1	2	3	4	5	6	7	8	9
H	A	V	E				F	U	N

The hash codes for each of the strings are the following:

$$h(\text{A}) = 38,\ h(\text{E}) = 20,\ h(\text{F}) = 97,\ h(\text{H}) = 49,\ h(\text{N}) = 27,\ h(\text{U}) = 57,\ h(\text{V}) = 42$$

In which order can the elements have been inserted into the hash table?

Check the box(es) ☑ with the correct alternative(s) – there can be more than one:

- ✓ V F U N H A E
- ☐ H A V E F U N
- ✓ F U N H A V E
- ☐ F V U N H E A
- ☐ F V N U H A E

Now, pick one of the correct alternatives and insert the elements in ***reversed order*** into the following linear probing hash table:

0	1	2	3	4	5	6	7	8	9
E	U	V	F				N	A	H

In which order did you insert the elements? E V A H N U F

Alternatively:

0	1	2	3	4	5	6	7	8	9
E	U	F	V				N	A	H

In which order did you insert the elements? E A H N U F V

Basic question 6: Quicksort partitioning

Perform a quicksort partitioning in-place of the following array:

0	1	2	3	4	5	6	7	8
17	97	21	34	37	1	13	42	33

Use the element at index 3 (i.e., the number 34) as pivot, and then partition. Recall that the partitioning algorithm also swaps the pivot into the correct place afterwards.

Note: quicksorting the left and right parts of the partition is not part of this question.

Write down how the array looks after the partitioning:

0	1	2	3	4	5	6	7	8
1	33	21	17	13	34	37	42	97

What sequence of swaps did you make when partitioning the array?

First we swap the pivot with the first element: **17—34**

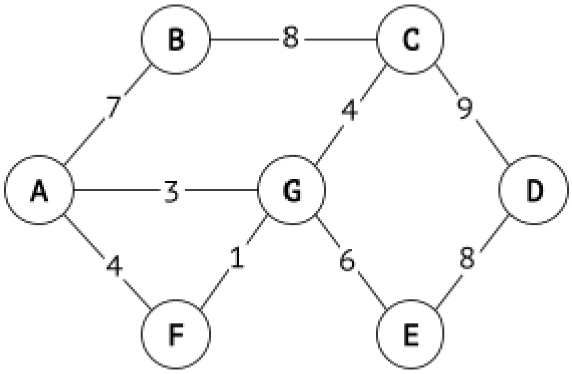
Then we partition: **97—33, 37—13**

Finally we swap the pivot in place: **34—1**

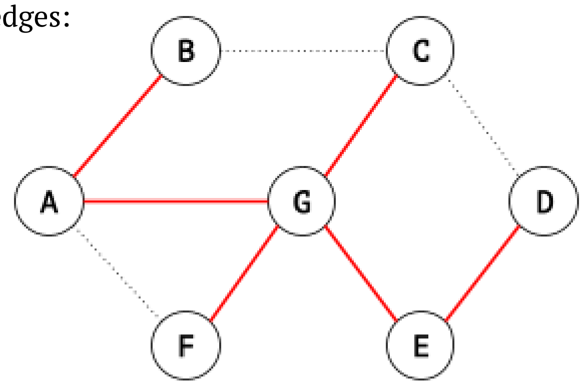
If you used a different algorithm from that of the course, explain it here:

Basic question 7: Graphs

Here is a weighted undirected graph:



Perform **Prim's algorithm** to construct the minimum spanning tree (MST) for the graph. Draw the MST by filling the dotted edges:



List the edges of the MST *in the order they are produced* by the algorithm. Do this for two different starting vertices.

- Write the edges in the form AC, DF, …
- You do not have to use all rows.

Starting vertex: A

1	AG
2	GF
3	GC
4	GE
5	AB
6	ED
7	

Starting vertex: D

1	DE
2	EG
3	GF
4	GA
5	GC
6	BA
7	

Basic question 8: Complexity

Here is a sorting algorithm that uses a minimum priority queue for sorting an array of numbers.

```
def sort(array):
    S = new empty binary heap

    for each x in array:
        insert x into S

    i = 0
    while S is not empty:
        y = delete minimum from S
        array[i] = y
        i += 1
```

What is the worst-case asymptotic time complexity in the size n of `array`? Answer for each of the two loops, and the total complexity. Write your answer in O-notation, and be as exact and simple as possible.

Complexity of the for loop: O($n \log(n)$)

Complexity of the while loop: O($n \log(n)$)

Complexity of the function sort: O($n \log(n)$)

Brief explanation:
(Alternatively, you can add comments directly to the code above if you want.)

The for loop is iterated $O(n)$ times, so the maximum size of the heap S will be $O(n)$. Therefore inserting into S is $O(\log(n))$. So the for loop has complexity $O(n) \cdot O(\log(n)) = O(n \log(n))$.

The while loop is iterated once for every element in S, so it is iterated O(n) times. Deleting from S is $O(\log(n))$, and array assignment is constant time, which means that the body has complexity $O(\log(n)) + O(1) = O(\log(n))$. So the while loop has complexity $O(n) \cdot O(\log(n)) = O(n \log(n))$.

The loops are performed in sequence, meaning that the final complexity is the same, $O(n \log(n))$.